A Triple-Through Method for Characterizing Test Fixtures

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Abstract —**Test fixtures for evaluating microwave components such as transistors or MMIC's consist of two "unmeasurable" sections, each having, for example, one coax and one microstrip terminal. A method is proposed for evaluating the S parameters of these sections through three conventional reflection/transmission measurements. It rests on the use of an auxiliary 2-port. No microstrip standard is needed, except for a load that is necessary if the SWR of the auxiliary 2-port is not low enough.**

I. Introduction

MANY METHODS have been proposed for removing the effect of the test fixture used in component measurements, a survey being given in [1]. Among them, the one-tier methods directly include the test fixture effect in the network analyzer error model, allowing real-time display of the device S parameters [2]. If the S parameters of the test fixture are to be known separately (as for noise measurements), then only two-tier methods work. With these methods, the network analyzer is first calibrated using coax standards. The test fixture parameters are then derived from various measurements. In the most classical case, three reflection measurements are performed on each part of the test fixture, with successively a short, an open, and a load connected to the microstrip terminal. The open, short, delay method [3] is based on two reflection measurements with an open and a short and one reflection/transmission measurement with a delay line inserted between the two test fixture parts. Finally, the short, through, delay method [4] uses one reflection measurement with a short and two reflection/transmission measurements with a through connection and a delay inserted. In this paper, a method is presented based on three reflection/transmission measurements that minimizes the microstrip hardware, as only a load is needed or even no standard at all. The method is directly adaptable to other environments such as waveguide and finline.

II. Principle

Suppose (Fig. 1) we have a test fixture consisting of two sections A and B, each including for example a coax-to-microstrip transition, bias circuitry, and so on. No assumption is made about the sections. Particularly, they need not be identical, as in [5]. We further suppose an auxiliary 2-port C is available that also has one coax and one

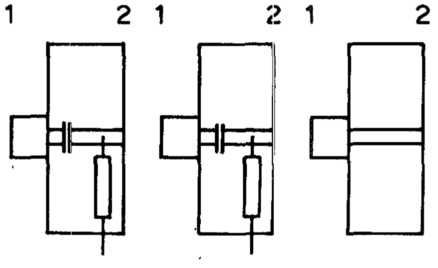

Fig. 1. The three 2-ports under use; A and B are the input and output sections of the test fixture to be characterized; C is an auxiliary 2-port (for example a coax to microstrip transition).

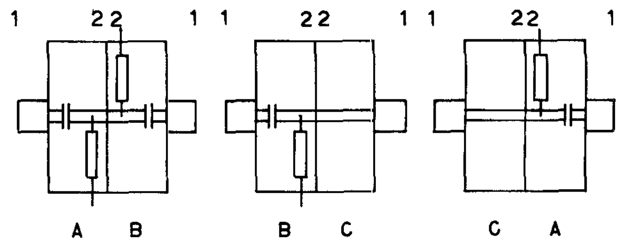

Fig. 2. The three 2-ports, obtained by through-connecting A, B and C, that are to be measured.

microstrip port. All three of these 2-ports are assumed to be reciprocal, a condition easily met by test fixtures. In every case the coax port is called no. 1 and the microstrip port no. 2. We now perform three S-parameter measurements on the 2-ports in Fig. 2, obtained by cascading and through-connecting 2-ports A, B, and C. Note that the resulting 2-ports have two coax ports and can as a consequence be accurately measured on a network analyzer calibrated with coax standards. Let us denote by

$$T_{AB}, T_{BC}, T_{CA}$$

the wave transfer matrices in the forward direction (from the left to the right) that are derived from these measurements. Let us further call

$$T_A, T_B, T_C$$

the wave transfer matrices of the 2-ports A, B, C in the coax-to-microstrip direction, and

$$T'_A, T'_B, T'_C$$

the corresponding matrices in the reverse direction. The relation between the T and T' matrices, calculated in the

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Appendix, is

$$T' = I'T^{-1}I' \tag{1}$$

where I' is defined by

$$I' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Also note that

$$I'^{-1} = I'. \tag{2}$$

As a consequence, the three measurements we perform can be expressed as

$$T_{AB} = T_A T'_B \tag{3}$$

$$T_{BC} = T_B T'_C \tag{4}$$

$$T_{CA} = T_C T'_A. \tag{5}$$

Each 2-port having three independent S parameters, it could be thought that the nine equations that follow when the above system is converted back to S parameters would make it possible to completely characterize the text fixture. However, on physical grounds, it seems unreasonable that the impedance level at the microstrip side could be found without any reference at this terminal. A numerical verification confirmed this: in fact equations (3) to (5) are not independent. So, something more has to be known. This feature will clearly appear when the system is solved. To this purpose, the following procedure is suggested.

First replace the T' matrices by their value (1) in (3) to (5), yielding

$$T_{AB} = T_A I' T_B^{-1} I' \tag{6}$$

$$T_{BC} = T_B I' T_C^{-1} I' \tag{7}$$

$$T_{CA} = T_C I' T_A^{-1} I'. \tag{8}$$

Then rewrite successively (6), (7), and (8) as

$$T_A = T_{AB} I' T_B I' \tag{9}$$

$$T_B = T_{BC} I' T_C I' \tag{10}$$

$$T_C = T_{CA} I' T_A I'. \tag{11}$$

Finally substitute in order to obtain

$$T_C = T_{CA} I' T_{AB} I' T_{BC} I' T_C I'$$

or

$$T_C I' T_C^{-1} I' = T_{CA} I' T_{AB} I' T_{BC}. \tag{12}$$

The first member of (12) can be viewed as the transfer matrix of the back to back cascade connection of 2-port C with itself (Fig. 3). The result is obviously a symmetrical 2-port. The second member of (12) is known, as it is a function of the measured transfer matrices T_{AB}, T_{BC}, and T_{CA}. Going back to the S parameters, let us denote by

$$\begin{bmatrix} S_R & S_T \\ S_T & S_R \end{bmatrix}$$

the S-parameter matrix of the 2-port in Fig. 3 and by

$$\begin{bmatrix} S_1 & S_t \\ S_t & S_2 \end{bmatrix}$$

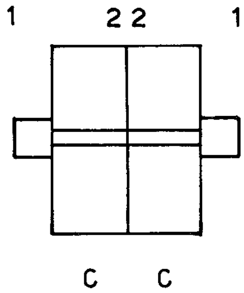

Fig. 3. The fictitious 2-port described by (12).

the S matrix of the auxiliary 2-port C. Using the S-parameter formulas for cascaded networks, system (12) can be written as

$$\begin{bmatrix} S_1 + S_t^2 S_2/(1-S_2^2) & S_t^2/(1-S_2^2) \\ S_t^2/(1-S_2^2) & S_1 + S_t^2 S_2/(1-S_2^2) \end{bmatrix} = \begin{bmatrix} S_R & S_T \\ S_T & S_R \end{bmatrix}. \tag{13}$$

From it follow the two equations

$$S_1 + S_t^2 S_2/(1-S_2^2) = S_R \tag{14}$$

$$S_t^2/(1-S_2^2) = S_T. \tag{15}$$

Solving them for S_2 and S_t, we get

$$S_2 = (S_R - S_1)/S_T \tag{16}$$

$$S_t = \pm\sqrt{S_T\left(1 - \frac{(S_R - S_1)^2}{S_T^2}\right)}. \tag{17}$$

Equations (16) and (17) show that S_2 and S_t are obtained provided S_1 is known. This completes the determination of the S matrix of 2-port C. It is then a simple matter to find the T and S matrices of 2-ports A and B (the actual test fixture parts) from (10) and (11):

$$T_B = T_{BC} I' T_C I' \tag{18}$$

$$T_A = I' T_{CA}^{-1} T_C I'. \tag{19}$$

III. Discussion

As explained above, in order to solve the problem, the S_1 parameter has to be known. S_1 is the reflection factor of the auxiliary 2-port C with a load connected to its microstrip terminal. This means that such a load is in fact needed, or that S_1 has to be determined by some other means. However, remember that 2-port C can be chosen arbitrarily. For example, it may be simply a transition from coax to microstrip that can exhibit a very low SWR if properly designed [6]. Accordingly, even in the absence of a good microstrip load, S parameters for the test fixture are obtained whose accuracy is only limited by the quality (SWR) of the transition available. Note that the loss and phase shift of the transition do not introduce any error, as their effects are incorporated in (3)–(5). Moreover, once the auxiliary 2-port has been characterized, the S parame-

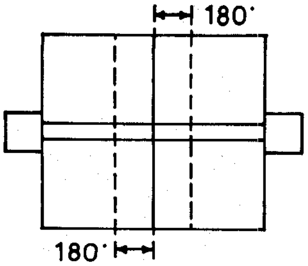

Fig. 4. Adding 180° electrical length to 2-port C gives the same overall transmission phase.

ters of further test fixtures are derived in only two steps: measuring 2-ports B + C and C + A (see (18) and (19)).

IV. Phase Uncertainty

A plus or minus sign appears in (17). It corresponds to a 180° phase shift and can be explained as follows: adding 180° to the phase of the transmission parameter S_t of 2-port C would give the same overall measurement S_T (compare Fig. 3 and Fig. 4). In order to choose the right sign and thus the right phase, the approximate electrical length l of 2-port C is needed. It can generally be computed from the physical dimensions. Using this parameter, we have

$$\arg S_t \approx -\beta l = -(2\pi f l)/c \tag{20}$$

where

- β propagation factor,
- f frequency,
- c velocity of light.

At each frequency the phase of S_t that best fits in with the theoretical value should be chosen. Note that phase uncertainty is not specific to this method. For example, the same problem arises when classically calibrating a test fixture by connecting three impedances at the microstrip ports: a 180° phase shift does not modify the reflection factor and so remains indeterminate.

V. Experimental Verification

The main question that arises is: how accurate are the S parameters of the test fixture sections derived in this way? To answer it, the method was tested on three measurable (connectorized) 2-ports. Sections A and B of the test fixture were simulated by two commercial bias networks with PC-7 connectors. A PC-7 to N male and N female to PC-7 adapter combination was used as the auxiliary 2-port C. The S parameters of these 2-ports were determined:

a) directly on a network analyzer calibrated with PC-7 standards;
b) by the method described above, i.e., by measuring the four S parameters of the three cascades A + B, B + C, C + A and applying formulas (16) to (19).

The results are summarized in Table I. Part 1 of this table shows the S parameters for the 2-ports A + B, B + C, C + A. Part 2 contains the S parameters derived by the procedure described in this paper. Directly measured S parameters are given in part 3, while in part 4 the differences between derived and measured S parameters (error vectors) are calculated. It is seen that the agreement is quite good, no error vector being larger than $8 \cdot 10^{-3}$. Part 5 of the table is for the S parameters derived with the assumption $S_1 = 0$. The errors (see part 6) are now somewhat larger, but remain quite acceptable for practical purposes. Also note that the assumption $S_1 = 0$ essentially affects the reflection parameters, the transmission parameters remaining virtually unchanged. Accordingly it is expected that, under similar conditions, S parameters for "unmeasurable" test fixture parts are obtained to within 10^{-2}. Moreover, no frequency sensitivity has to be feared, as is the case with methods involving delay lines [1], making broad-band characterization possible.

TABLE I
Experimental Results for a Simulated Test Fixture Characterization
(Phases: Degrees; Frequency: 2 GHz)

	2-port	MagS11	ArgS11	MagS12	ArgS12	MagS21	ArgS21	MagS22	ArgS22
1	A+B	.021	-38.0	.978	-147.8	.975	-147.9	.023	-92.5
	B+C	.016	-1.8	.978	-79.7	.978	-79.4	.013	2.5
	C+A	.026	-18.0	.978	-79.8	.978	-79.5	.025	23.1
2	A	.028	12.8	.987	106.3	.987	106.0	.029	9.9
	B	.020	-11.8	.987	106.1	.987	106.4	.019	36.9
	C	.008	-115.7	.991	174.2	.991	174.2	.006	-75.3
3	A	.026	3.5	.984	105.9	.984	106.2	.026	13.0
	B	.020	-17.2	.984	106.0	.984	106.4	.018	30.4
	C	.008	-115.7	.994	174.4	.994	174.4	.008	-102.2
4	A	.0048	-106.3	.0075	-7.5	.0046	-122.8	.0033	165.1
	B	.0019	104.5	.0035	-44.1	.0030	-73.6	.0023	-81.8
	C	.0000	-	.0046	-136.6	.0046	-136.6	.0038	-147.9
5	A	.029	-2.8	.987	106.3	.987	106.0	.026	-6.4
	B	.025	-27.5	.987	106.1	.987	106.4	.013	14.1
	C	.000	-	.991	174.2	.991	174.2	.013	-92.5
6	A	.0043	135.1	.0075	-7.5	.0046	-122.8	.0088	93.3
	B	.0064	118.6	.0035	-44.1	.0030	-73.6	.0066	63.9
	C	.008	-115.7	.0046	-136.6	.0046	-136.6	.0053	102.3

1: basic triple-through measurements
2: S parameters derived
3: directly measured S parameters
4: error vectors (3–2)
5: S parameters derived with the assumption $S_1 = 0$
6: error vectors (3–5)

VI. Conclusions

A method for characterizing test fixtures has been described that essentially rests on three standard (coax) S-parameter measurements. The use of microstrip standards is reduced to a minimum, as only one reflection measurement with a load has to be performed on an auxiliary 2-port. It may even be canceled if this 2-port is a transition with low SWR. The method is broad-band, frequency insensitive, and extendable to other media.

Appendix

Suppose (Fig. 5) a linear reciprocal 2-port is described by its left-to-right transfer matrix:

$$\begin{bmatrix} B_1 \\ A_1 \end{bmatrix} = \begin{bmatrix} T_{ba} & T_{bb} \\ T_{aa} & T_{ab} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}. \tag{21}$$

Expressing the T parameters as a function of the S param-

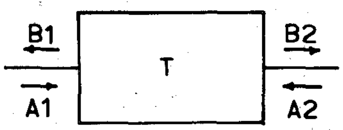

Fig. 5. Defining the symbols used in (21).

eters [7] yields

$$[T]=\begin{bmatrix} -\Delta S/S_{21} & S_{11}/S_{21} \\ -S_{22}/S_{21} & 1/S_{21} \end{bmatrix}$$

with

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}.$$

To obtain the S matrix of the reversed 2-port we have to interchange

$$S_{11} \leftrightarrow S_{22}.$$

Thus the left-to-right transfer matrix of the reversed 2-port is

$$[T']=\begin{bmatrix} -\Delta S/S_{21} & S_{22}/S_{21} \\ -S_{11}/S_{21} & 1/S_{21} \end{bmatrix}=\begin{bmatrix} T_{ba} & -T_{aa} \\ -T_{bb} & T_{ab} \end{bmatrix} \quad (22)$$

which can be written as:

$$T' = I'T^{-1}I' \quad (23)$$

with

$$[I']=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

References

[1] R. Lane, "De-embedding device scattering parameters," *Microwave J.*, vol. 27, no. 8, pp. 149–152, 154–156, Aug. 1984.

[2] G. E. Elmore and L. J. Salz, "Quality microwave measurement of packaged active devices," *Hewlett-Packard J.*, vol. 38, no. 2, pp. 39–48, Feb. 1987.

[3] R. L. Vaitkus, "Wide-band de-embedding with a short, an open and a through line," *Proc. IEEE*, vol. 74, pp. 71–74, Jan. 1986.

[4] J. Archer, "Implementing the TSD calibration technique," *Microwave Syst. News*, vol. 17, no. 5, pp. 54–63, May 1987.

[5] E. R. Ehlers, "Symmetric test fixture calibration," in *1986 IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 275–278.

[6] E. H. England, "A coaxial to microstrip transition," *IEEE Trans. Microwave Theory Tech.*, vol. 24, pp. 47–48, Jan. 1976.

[7] K. C. Gupta, R. Garg, and R. Chadha, *Computer-Aided Design of Microwave Circuits*. Dedham, MA: Artech House, 1981, p. 36.

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